Negative Exponents and Geometric Sequences - Review

A. Negative Exponents – $a^{-n} = \frac{1}{a^n}$ - always leave positive exponents

(a) $(m^3p^{-4}r^2)^{-6}$

$(m^{-18}p^{24}r^{-12})$

$\frac{p^{24}}{m^{18}r^{12}}$

(b) $(2c^4k^{-2})^{-3}(4c^{-5}k^3)^4$

$2^{-3}c^{-12}k^6 \cdot 4^4c^{-20}k^{12}$

$\frac{1}{8}c^{-32}k^{18} \cdot 256$

$\frac{256k^{18}}{8c^{32}} = \frac{32k^{18}}{c^{32}}$

(c) $\left(\frac{-3m_1^4p^{-2}t^8}{15m^{-6}p^3t^5}\right)^{-3} = \left(\frac{-1m^4 \cdot m^6t^8}{5p^3p^2t^5}\right)^{-3}$

$= \left(\frac{-1m^{10}t^3}{5p^5}\right)^{-3} = \frac{-1^{-3}m^{-30}t^{-9}}{5^{-3}p^{-15}}$

$= \frac{125p^{15}}{-1m^{30}t^9}$ or $-\frac{125p^{15}}{m^{30}t^9}$

B. Geometric/Arithmetic Sequences

* Is the sequence arithmetic, geometric or neither?
* Find the next 3 terms

(a) 4, -2, 1, $-\frac{1}{2}$, … $\frac{1}{4}$, $-\frac{1}{8}$, $\frac{1}{16}$

geometric

$r = -\frac{1}{2}$

(b) 1, 2, 3, 5, 8, 13, 21, 34, 55

Neither

(Fibonacci Sequence)

(c) 8, 5, 2, -1, -4, -7, -10

arithmetic

$d = -3$

C. Geometric Sequences

* Identify a_1, Find the common ratio, r …. $r = \frac{a_{n+1}}{a_n}$

(a) 5, 10, 20, 40, ….

$a_1 = 5$

$r = \frac{10}{5} = 2$

(b) 54, 9, $\frac{3}{2}$, ….

$a = 54$

$r = \frac{9}{54} = \frac{1}{6}$

(c) 72, -24, 8, ….

$a = 72$

$r = \frac{-24}{72} = -\frac{1}{3}$

* Write the explicit formula: $a_n = a_1(r)^{n-1}$

(a) -4, -12, -36, … $r = \frac{-12}{-4} = 3$

$a_n = -4(3)^{n-1}$

(b) $\frac{4}{5}$, 2, 5, $\frac{25}{2}$, …. $r = \frac{5}{2}$

$a_n = \frac{4}{5} \cdot \left(\frac{5}{2}\right)^{n-1}$

(c) 2, 8, 32, 128, …. $r = \frac{8}{2} = 4$

$a_n = 2(4)^{n-1}$

* Find a specified term

(a) Find a_{12}, given $a_1 = 2$, r = 5

$a_n = 2(5)^{n-1}$

$a_{12} = 2(5)^{12-1}$

$a_{12} = 2(5)^{11} = 97{,}656{,}250$

(b) Find a_9, given $a_1 = -20$, r = -3

$a_n = -20(-3)^{n-1}$

$a_9 = -20(-3)^{9-1}$

$a_9 = -20(-3)^8$

$a_9 = -131220$

$\frac{7}{8} \div \frac{1}{4} = \frac{7}{8} \cdot \frac{4}{1} = \frac{7}{2}$

* **Find missing terms**

(a) -9, $\underline{-18}$, $\underline{-36}$, $\underline{-72}$, -144

$\frac{-9 \cdot r^4}{-9} = \frac{-144}{-9}$ $r^4 = 16$ $\sqrt[4]{r^4} = \sqrt[4]{16}$ $r = 2$

(b) $\frac{20}{5} = \underline{4}$, 20, $\underline{100}$, $\underline{500}$, 2500

$\frac{20 \cdot r^3}{20} = \frac{2500}{20}$ $r^3 = 125$ $\sqrt[3]{r^3} = \sqrt[3]{125}$ $r = 5$

(c) $\underline{14}$, $\underline{\frac{7}{2}}$, $\frac{7}{8}$, $\underline{\frac{7}{32}}$, $\frac{7}{128}$

$\frac{7}{8} \cdot r^2 = \frac{7}{128}$

$\frac{8}{7}\left(\frac{7}{8}r^2\right) = \left(\frac{7}{128}\right)\frac{8}{7}$

$r^2 = \frac{1}{16}$

$\sqrt{r^2} = \sqrt{\frac{1}{16}}$ $r = \frac{1}{4}$

$\frac{7}{2} \div \frac{1}{4} = \frac{7}{2} \cdot \frac{4}{1} = 14$

* **Find a_1**

(a) $a_4 = 108$, r = 3

$a_4 = 108$, $n = 4$

$a_n = a_1(3)^{n-1}$

$108 = a_1(3)^{4-1}$

$108 = a_1(3)^3$

$\frac{108}{27} = \frac{a_1 \cdot 27}{27}$

$a_1 = 4$

(b) $a_5 = 20$, $a_9 = 320$ ($n = 5$)

20, __, __, __, 320

$\frac{20 \cdot r^4}{20} = \frac{320}{20}$

$r^4 = 16$

$r = 2$

$a_n = a_1(2)^{n-1}$

$20 = a_1(2)^{5-1}$

$20 = a_1(2)^4$

$\frac{20}{16} = \frac{a_1(16)}{16}$

$\frac{5}{4} = a_1$

(c) $a_2 = -24$, $a_5 = 1536$ ($n = 2$)

-24, __, __, 1536

$\frac{-24 \cdot r^3}{-24} = \frac{1536}{-24}$

$r^3 = -64$

$\sqrt[3]{r^3} = \sqrt[3]{-64}$

$r = -4$

$a_n = a_1(-4)^{n-1}$

$-24 = a_1(-4)^{2-1}$

$\frac{-24}{-4} = \frac{a_1(-4)}{-4}$

$6 = a_1$

or $a_2 = a_1 \cdot r$

$-24 = a_1(-4)$

* **Application problems**

You are reducing a copy of a photograph. The length of the photo is 10 inches. The smallest size a copier can make is 64% of the original.

10, 6.4, 4.096, ...

(a) Write the explicit formula for this scenario.

$a_n = 10(0.64)^{n-1}$

(b) Find the length of the photo after 5 reductions.

$n = 6$ (first reduction = 2nd term)

$a_6 = 10(0.64)^{6-1} = 10(0.64)^5 = 1.074$ inches

(c) How many reductions will it take for the length of the photo to be less than 0.25 inches?

5th reduction 1.074 in

6th reduction $= 1.074 \cdot (0.64) = 0.687$ inches

7th reduction $= 0.687 \cdot (0.64) = 0.440$ inches

8th reduction $= 0.440(0.64) = 0.281$ inches

9th reduction $= 0.281 \cdot (0.64) = 0.180$ inches

Will take 9 reductions

* Review all notes and homework.